

INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT First Year
2016-17 Semester II

Design of Experiments
Final Examination

29 April 2017

Duration: 3 hours

Note: • Notations and symbols as used in the class are followed. • Answer as many questions as possible, but the maximum you can score from these questions is 58. • Points are indicated at the end of each question.

1. Prove that a connected block design is variance - balanced if and only if its C -matrix has all diagonal elements equal and all the off diagonal elements equal. [12]
2. Show that for an equireplicate, binary, proper and variance - balanced design, the incidence matrix N satisfies the following .

$$N'N = (r - \lambda)I + \lambda JJ'$$

where λ is a scalar such that $\lambda = \frac{r(k-1)}{a-1}$ and $J = (1, 1, \dots, 1)'$. [12]

3. Write brief notes on different types of optimality concepts used in block designs like A - optimality, D - optimality and E - optimality. [6]
4. Construct a BIBD with the following parameters $a = 11, k = 5$ and $\lambda = 2$. [7]
5. For the following BIBD recover the intrablock information.

Treatments	Blocks			
	1	2	3	4
1	73	74	-	71
2	-	75	67	72
3	73	75	68	-
4	75	-	72	75

Compare the combined estimates τ^* with the intrablock estimates, $\hat{\tau}$.
[7 + 7 + 1 = 15]

6. Verify whether the following statements are true or false. Justify your answers briefly.
- (a) A block design is said to be orthogonal if Q_i 's and P_j 's are orthogonal for all i and j . [2]
 - (b) If C -matrix of a block design contains a row (column) of nonzero elements, then the design is connected. [2]
 - (c) Suppose there exists a BIBD with parameters (a, k, λ_1) and another BIBD with parameters (a, k, λ_2) . Then there exists a BIBD with parameters $(a, k, \lambda_1 + \lambda_2)$. [2]
 - (d) A 2^k design is D -optimal design for fitting the first-order model or the first order-model with interaction. [2]
7. An engineer is interested in studying the effects of cutting speed (A), tool geometry (B) and tool angle (C) on life in hours of a machine tool. Two levels of each factor are chosen and three replicates of a 2^3 factorial design are run.
- (a) Explain what is meant by main effects and interaction effects of factors. Discuss how one can estimate them.
 - (b) If the above experiment is to be run in two blocks, assign treatment combinations into blocks such that the higher order interaction ABC is confounded with blocks. [6 + 4 = 10]